Analysis of Stress at Several Junctions in Pressurized Shells

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Theoretical and experimental results are presented for the discontinuity stresses arising at a change of wall thickness in a cylinder, a cylinder-hemisphere junction, and a cone-spherical torus junction in pressure vessels. The effect of mismatch of nonconcurrence of the middle surfaces of two joined cylinders is considered. In addition, a cylinder with a special closure which has considerably reduced stresses is described, and curves with theoretical and experimental stresses are presented.

THE discontinuity stresses that arise in pressure vessels due to changes in geometry or thickness are of a local nature, but they can be of appreciable magnitude. Ref. 1 presents a theoretical treatment of the discontinuity stresses occurring in a wide range of shells. This note discusses the experimental verification of the stress states existing at four different shell junctions shown in Fig. 1.

Change of Thickness in Cylinder

At a change of thickness in a cylindrical pressure vessel, the discontinuity stresses are a result of two distinct conditions: the difference in radial expansion of the two regions of the cylinder due to the different wall thicknesses, and the possible mismatch of the middle surfaces of the two regions due to a difference in mean radii. A complete discussion of this problem is presented in Ref. 2.

Cylinders with changes in wall thickness were investigated experimentally for the following three cases: 1) continuous middle surface, 2) continuous inner surface, and 3) continuous outer surface. These specimens were machined from 6-in. nominal diameter 2014-T6 extruded aluminum tubing. The theoretical and experimental stress distributions for these cases are shown in Fig. 2. Because the nominal ratio of diameter to thickness in the thicker region of these cylinders was about 28, the membrane stresses were computed using thick-wall equations. The experimental and theoretical distributions of stress are generally in good agreement. It will be noted that the longitudinal stress on the inner surface of the thinner region of the cylinder peaks considerably above the maximum membrane stress for the case with a continuous outer surface. It is true, however, that this stress decays very rapidly to a more reasonable value.

Toriconical Head

One of the cylinder end closures investigated was a toriconical head in which the torus was a portion of a sphere (see Figs. 1b and 1c). This closure contains two discontinuities, namely, a junction between a cylinder and a portion of a sphere and a junction between a cone and a portion of a sphere. The distance between the discontinuities is sufficient to allow them to be treated separately. The junction of the cylinder with a portion of a sphere is identical to the study of a cylinder with a hemispherical dome, and thus the classical treatment of this problem presented in Ref. 3 was used to determine the theoretical stress distribution in this region. The other discontinuity present in the toriconical head is the junction between the cone and the spherical torus. The analysis for the stress distribution in this region is taken from Refs. 1 and 4.

The toriconical test specimen with a 29.25-in. nominal

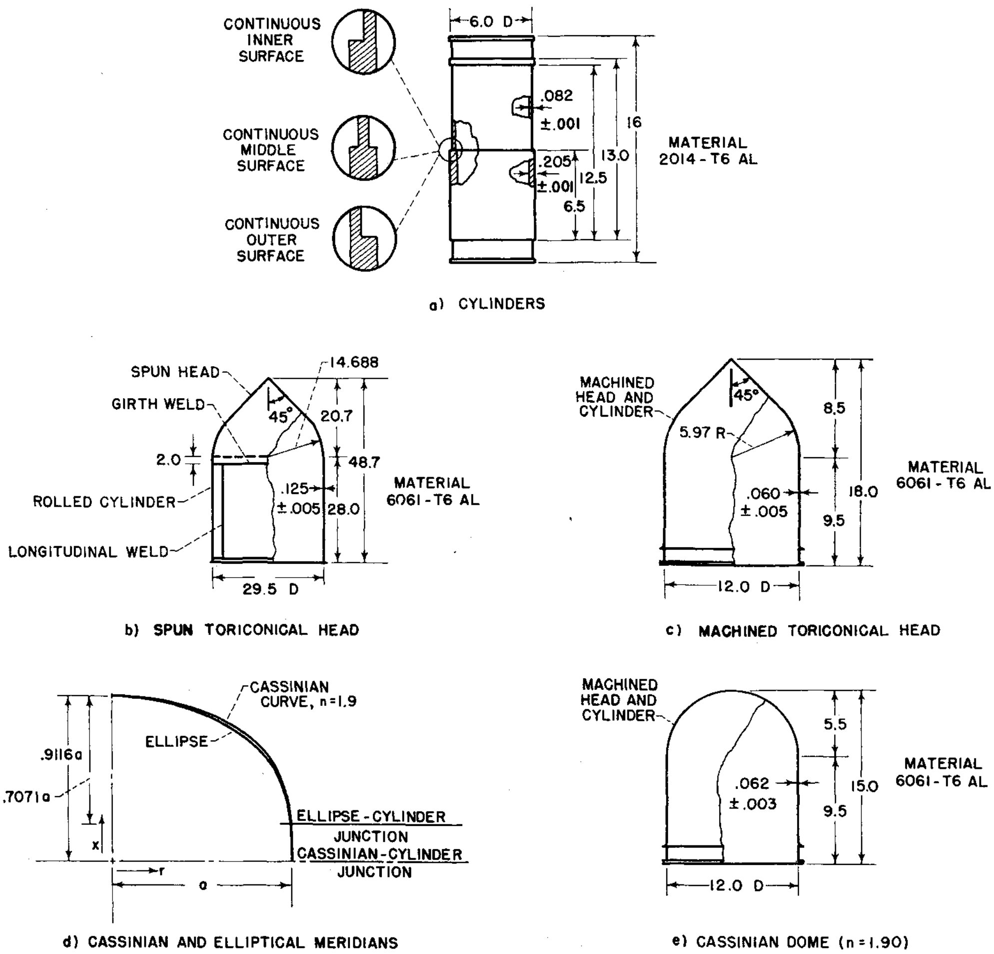

Fig. 1 Geometry of test specimens

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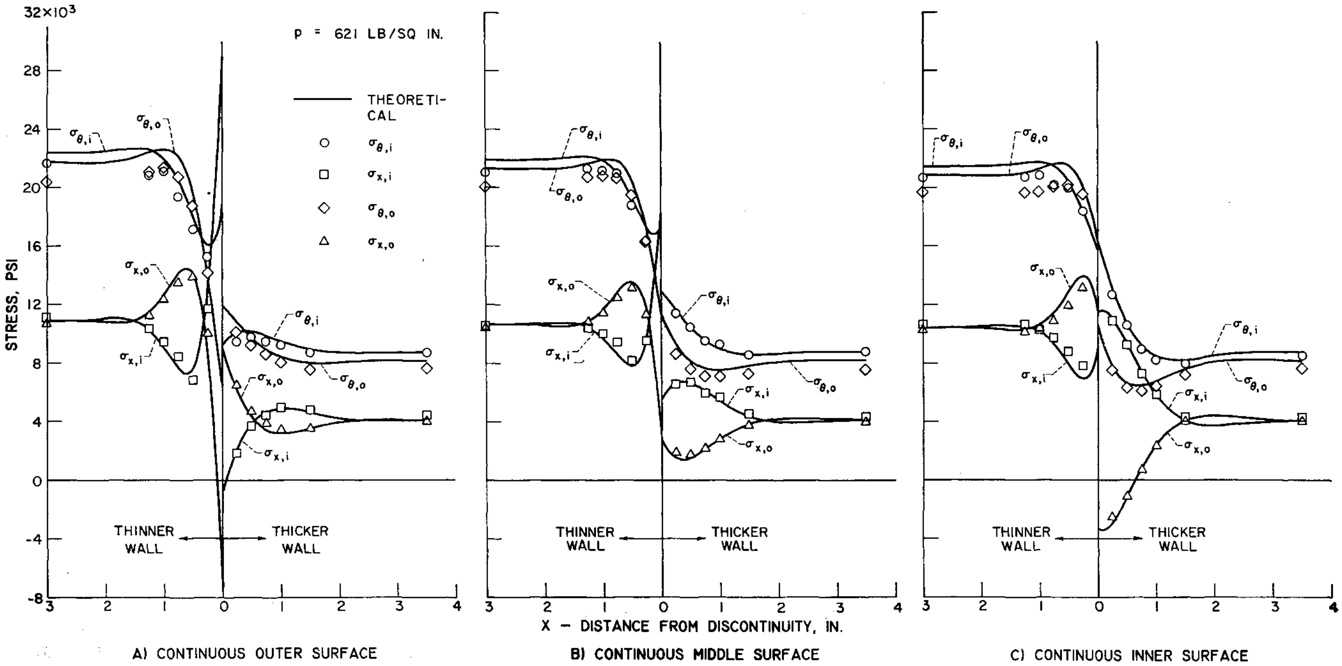

Fig. 2 Theoretical and experimental stress in cylinders

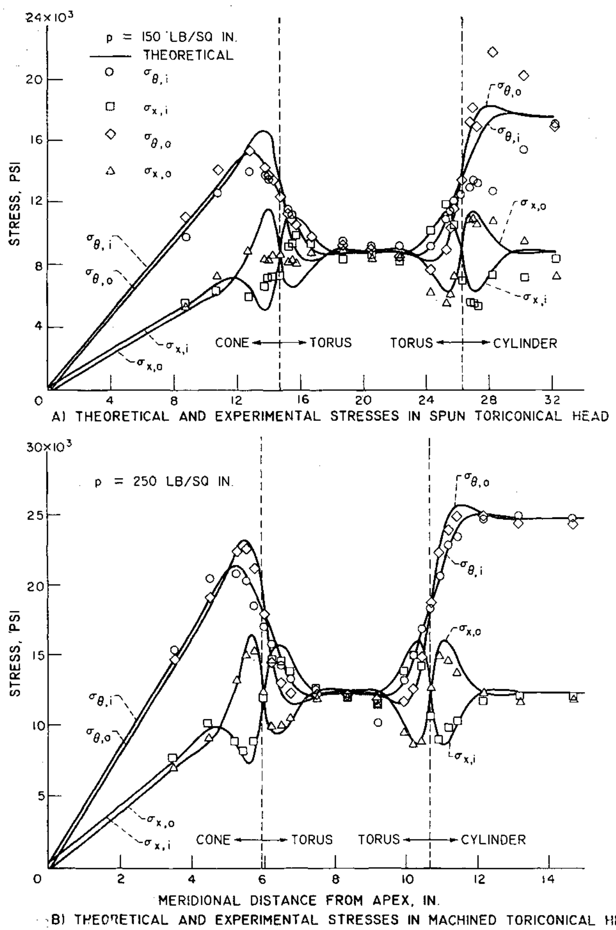

Fig. 3 Stresses in toriconical heads

mean diameter was spun from 0.125 sheet and then welded to a rolled cylinder as shown in Fig. 1b. Theoretical curves of stress distribution along with the experimental data points are shown in Fig. 3a for this shell. The trends of the experimental data are consistent with the theoretical curves. The discrepancy between the experimental and theoretical results is probably due primarily to fabrication errors in the test specimen and is probably typical of what generally can be expected in a spun dome of similar design. Note that the maximum experimental stress is considerably higher than the maximum stress computed theoretically.

A similar test model was contour-machined from a 6061-T6 aluminum billet (Fig. 1c). The nominal outer diameter of this specimen was 12 in., and the nominal thickness was 0.060 in. Curves of the stress distribution for this specimen are plotted in Fig. 3b. Agreement between the experimental and theoretical results is seen to be very good. The correlation in this instance is much better than it was for the 29.25-in.-diam spun toriconical head previously discussed. This is primarily because the desired geometry was obtained more nearly by machining than by spinning.

Cassinian Dome

A special cylindrical pressure vessel closure has been suggested in Ref. 5 and will be referred to hereafter as a Cassinian dome, because the equation of the meridian is a modified curve of Cassini. If the meridional radius of curvature is infinite at the function with the cylinder, the equation of the meridian is

$$(n^2x^2 + r^2)^2 - 2a^2(n^2x^2 - r^2) = 3a^4$$

The parameter n can be varied to give a family of curves. As n increases, the curves become flatter. If n is approximately 2, the curve assumes a shape that becomes reasonable for a pressure vessel bulkhead.

As can be seen in Fig. 1d, the Cassinian dome is very similar in shape to an ellipsoidal dome. One of the most desirable properties of this closure is that the radius of curvature of the meridian approaches infinity as the curve approaches the junction with the cylinder. Therefore, no sudden change of curvature takes place, and the bending stresses are considerably reduced.

A small cylinder with a Cassinian dome was contour-machined from a 6061-T6 aluminum billet (Fig. 1e). The nominal outer diameter of the cylinder was 12 in., and the nominal thickness was 0.062 in. Curves of the theoretical and experimental stresses for this specimen are shown in Fig. 4a. Reasonable agreement is obtained between the theoretical

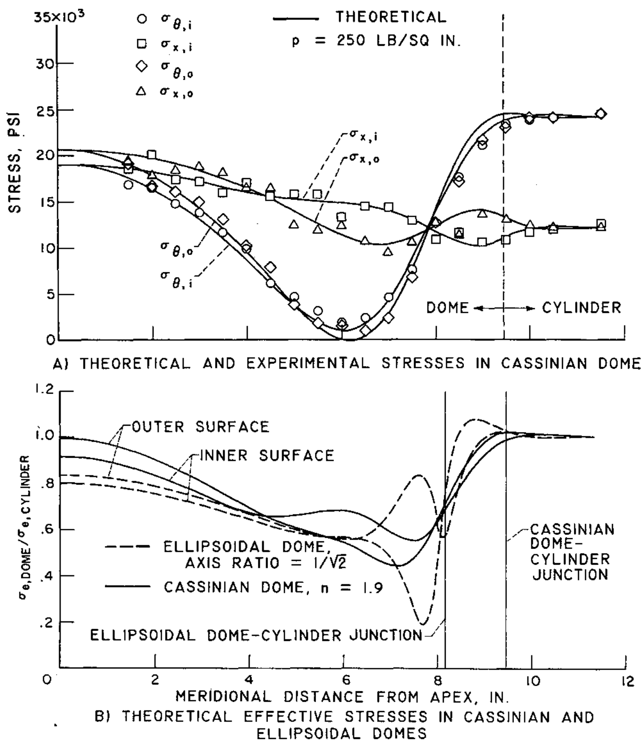

Fig. 4 Stresses in Cassinian and ellipsoidal domes

and experimental results. The theoretical stresses were computed using a numerical solution[6] of the Reissner equations[7] for the analysis of thin shells of revolution.

The Cassinian dome used for the experimental stress analysis had a parameter n of 1.90. This value was selected because it is the flattest head of this nature which has no compressive membrane stresses. Thus, circumferential buckling problems are eliminated. The flattest ellipsoidal dome that has no compressive membrane stress has a semiminor to semimajor axis ratio of 0.707. This ellipsoidal dome is compared geometrically with the Cassinian dome just described in Fig. 1d. It can be seen that the two curves are very similar in shape, although the depth of the Cassinian dome is larger than that of the ellipsoidal dome. However, because the Cassinian dome is nearly cylindrical over a large portion of its depth, interstage structures between propellant tanks with this type of dome probably would not extend from cylinder to cylinder. They would be more nearly the same length as those for tanks with ellipsoidal domes such as the one shown. Also, the volume of a propellant tank with Cassinian domes such as the one just described is slightly greater than the volume of a tank of equal length but having ellipsoidal domes with an axis ratio of 0.707.

The stresses were computed for a pressure vessel having the same outer radius and thickness as the Cassinian tank but with an ellipsoidal dome having an axis ratio of 0.707. Using the distortion energy theory, the effective stresses for this shell are compared to those for the shell with a Cassinian dome in Fig. 4b. The maximum effective stress ratio arising from the ellipsoidal dome is 1.076 vs 1.017 for the cylinder with the Cassinian dome. Thus, the yield pressure when using the Cassinian dome is about 6% higher than that obtained when using a comparable ellipsoidal dome. There is also considerably less bending in the knuckle region of the Cassinian dome, although the stresses near its apex are considerably higher but not critical.

References

1 Johns, R. H. and Orange, T. W., "Theoretical elastic stress distributions arising from discontinuities and edge loads in several shell-type structures," NASA TR R-103 (1961).

2 Morgan, W. C. and Bizon, P. T., "Experimental investigation of stress distributions near abrupt change in wall thickness in thin-walled pressurized cylinders," NASA TN D-1200 (1962).

3 Timoshenko, S. and Woinowsky-Krieger, S., *Theory of Plates and Shells* (McGraw-Hill Book Co. Inc., New York, 1959), 2nd ed., p. 482.

4 Taylor, C. E. and Wenk, E., Jr., "Analysis of stress in the conical elements of shell structures," Rept. 981, David W. Taylor Model Basin (May 1956).

5 Flügge, W., "Stress problems in pressurized cabins," NACA TN 2612 (1952).

6 Radkowski, P. P., Davis, R. M., and Bolduc, M. R., "A numerical analysis of the equations of thin shells of revolution," ARS Preprint 1580-60 (1960).

7 Reissner, E., "On the theory of thin elastic shells," *Reissner Anniversary Volume* (Edwards Brothers Inc., Ann Arbor, Mich., 1949), pp. 231–247.

Performance of an Electromagnetic Actuation System

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Nomenclature

H_y	= component of the earth's field along the longitude line, oe
H_z	= component of the earth's field along the local geocentric vertical, oe
M_e	= magnetic moment of the earth, unit-pole/cm
θ_L	= latitude, deg
r	= distance from earth's center, cm
T	= torque, dyne-cm
B	= earth's field strength, gauss
I	= current amperes
N	= number of turns
A	= area of coil, cm²
θ	= angle between earth's field and coil field, deg
M_x, M_y, M_z	= components of momentum, dyne-cm-sec
I_{cx}, I_{cy}, I_{cz}	= coil currents, amp

I. Introduction

ATTITUDE control systems may be classified into two general categories, depending upon the type of actuation system employed. One type uses a mass dispensing torque generation system such as compressed gas or chemical propellant, and a second type manipulates the natural forces of the space environment such as aerodynamic pressure, solar pressure, earth's mass attraction, and magnetically coupled forces. Mass dispensing systems have limited life due to fuel storability and stability problems, leakage, or the normal consumption of the fuel. The use of the majority of natural forces such as gravity gradient, solar pressure, and some forms of magnetic coupling provides very low torque levels; thus, mechanizations using these forces are limited. One method of magnetic coupling, however, provides comparatively high torque levels and a long life without restricting the attitude of the vehicle.

This method uses three mutually perpendicular coils for implementing momentum transfer and three mutually perpendicular inertia wheel controls for storing momentum and generating desired angular motions. Momentum transfer to the earth's field is accomplished by measuring the unwanted momentum stored in the inertia wheels with tachometer

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